Optimal Algorithms for Computing of "Minimum Width Annulus" with Rectilinear and Chebyshev distances and in Undirected Networks

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1 Extended Abstract

For a set of n points in $\mathbb{R}^2$ the *minimum width annulus problem* in the Euclidean space is motivated from statistical analysis, computational metrology, quality control for production process, pattern recognition, etc. (see [1], [2], [4], [8]). In location theory for the non-obnoxious set of location problems the objective function to be optimized is often of a minisum (minimizing of transportation costs) or minimax (minimizing of social costs) form. In many settings, especially in the public sector, the aspect of fairness is considered, which we model as an *equity problem*: we have n existing facilities and we want to place a new one so that a difference between maximal and minimal effects on existing facilities (we define an effect as the distance between old and new facilities) is minimal [6]. For Euclidean distances this *equity problem* is equivalent to the problem of computing the minimum width annulus for the given set of points.

However, the rectilinear distance l_1 is more appropriate for a certain class of problems (in urban settings, in robotic) than Euclidean distance, since for many applications this metric gives a better estimation of actual travel than the Euclidean metric. Solving of the *equity problem* in $\mathbb{R}^2_{l_1}$ leads to necessity of introducing of annulus concept in this space. Also minimum width annulus problem with rectilinear (or Chebyshev) distance can be useful for testing of "squareness" of a point set, for fitting a set of points by a diamond (square), for quality control for production process, for pattern recognition, etc.

Most of the network location models deal with efficiency (median) and effectiveness (center) measures. However, for public sector applications the consideration of equity aspects become important. Therefore introduction of minimum width annulus concept as one of the equity measures [6] in networks is proved from the practical point of view. To our knowledge minimum width annulus (equity) problem in $\mathbb{R}^2$ for rectilinear l_1 and Chebyshev l_∞ distances and in networks have not been studied so far. However the works of Kariv and Hakimi

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[5], Mesa et al. [7], Burkard and Dollani [3] have relation to the minimum width annulus problem in networks.

In this paper it is shown that the minimum width annulus problem in $\mathbb{R}^2_{l_1}$ is equivalent to *the rectilinear 1-center problem* in the sense that both have at least one common optimal point. This is not true for the Euclidean distance. We propose a $O(n)$ algorithm for the rectilinear 1-center problem which was extended to a new linear time algorithm for solving the minimum width annulus (unweighted equity) problem in $\mathbb{R}^2_{l_1}$. This algorithm can be used to find an optimal solution for the equity problem in $\mathbb{R}^2_{l_\infty}$ with the help of a transformation lemma. We present an exact algorithm for solving the minimum width annulus problem in undirected networks with running time $O(nm)$ and extend the problem to the weighted case resulting in a $O(n^2m)$ time algorithm (n is a number of vertices, m-edges in the network). Moreover, the algorithms for solving the minimum width annulus problems also solve the *1-circle location problem* in $\mathbb{R}^2_{l_1}$ and $\mathbb{R}^2_{l_\infty}$ in linear time and in undirected networks for the unweighted and weighted case with complexity $O(nm)$ and $O(n^2m)$, respectively .

Keywords: Equity location; Center problem; Minimum Width Annulus; Circle location

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