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Open & closet sets

SOLUTIONS

Problem 1

If A is a closed set that contains every rational number, $r \in [0,1]$, show that $[0,1] \subset A$.

A is closed, $r \in A$ $\qquad$ $\mathbb{R} \setminus A$ open

$[0,1] \subset A$

so, $r \in [0,1] \Rightarrow r \in A$

$y \in [0,1]$ & $y \neq$ rational

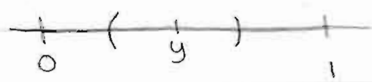
want: $y \in A$

assume by way of contradiction that $y \notin A$

$$y \in \underbrace{\mathbb{R} \setminus A}_{\text{open set}}$$

then, there exists an open interval I about y, $y \in I$ & $I \subseteq \mathbb{R} \setminus A$

I has no rational numbers



However, that is a contradiction as <u>any</u> open interval about y contains rational numbers

$\therefore$ $y \in A$.

Problem 2

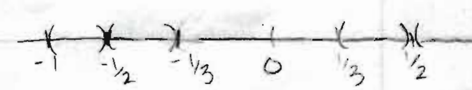
a) $\{x \in \mathbb{R}^n : \|x\| < 1\}$

I : $\|x\| < 1$ (set itself)

E : $\|x\| > 1$

B : $\|x\| = 1$

b) The collection of open intervals, $\left(\frac{1}{n}, \frac{1}{n+1}\right)$: $n \in \mathbb{Z}$



Let's call this set B.

I : B (the set itself)

E : $(-\infty, -1) \cup (1, \infty)$

B : $\{-1, -\frac{1}{2}, -\frac{1}{3}, \dots, 0, \dots, \frac{1}{3}, \frac{1}{2}, 1\}$

Problem 3

Find an open cover for.

Definition:
a collection θ of open sets is an open cover of A (or, briefly, covers) if every point $x \in A$ is in some open set in the collection θ

a) The closed interval $[-1, 1]$

b) The set of all rational numbers

$(0,0) \cup (-1,1) \cup (-2,2) \cup (-3,3) \cup (-4,4) \cup \dots \cup (-n,n)$