Norm and Inner Product Review Solution

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① Since x and y are perpendicular, $\langle x, y \rangle = 0$

$\Rightarrow x_1y_1 + x_2y_2 + \cdots + x_ny_n = \sum_{i=1}^{n} x_iy_i = 0.$

I show $\|x+y\|^2 = \|x\|^2 + \|y\|^2$.

$\|x+y\|^2 = \sum_{i=1}^{n}(x_i+y_i)^2$ by definition of norm

$= \sum_{i=1}^{n}(x_i^2+y_i^2) + 2\sum_{i=1}^{n}x_iy_i = \sum_{i=1}^{n}(x_i^2+y_i^2) + 2\cdot 0$ since $\sum_{i=1}^{n}x_iy_i = 0$ from above.

$= \sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2$ by property of Σ.

$= \|x\|^2 + \|y\|^2$ by definition of norm ∎

② $\|x\| = \sqrt{\sum_{i=1}^{n} x_i^2}$ by definition of norm.

$= \sqrt{\langle x, x \rangle}$ by definition of inner product ∎

③ Consider $\|a\|$. By the triangle inequality,

$\|a\| = \|a-b+b\| \leq \|a-b\| + \|b\|$ subtract $\|b\|$ from both sides

$\Rightarrow \|a\| - \|b\| \leq \|a-b\|$

Similarly

$\|b\| = \|b-a+a\| \leq \|b-a\| + \|a\|$

$\Rightarrow \|b\| - \|a\| \leq \|b-a\|$

Notice that $\|b-a\| = \|a-b\|$, and also $\|\,\|a\| - \|b\|\,\| = \|\,\|b\| - \|a\|\,\|$

$\therefore \|\,\|a\| - \|b\|\,\| \leq \|a-b\|$