# Solutions Extra Practice Problems

## Midterm 1 Math 225

1. Given the set $A = [0, 2) \cup (2, 3)$ in $\mathbb{R}$. Find
   (a) interior of $A$
   **Answer:** $(0, 2) \cup (2, 3)$
   (b) exterior of $A$
   **Answer:** $(-\infty, 0) \cup (3, \infty)$
   (c) boundary of $A$
   **Answer:** $\{0, 2, 3\}$

2. Given the set $A = [0, 2) \cup (2, 3)$ in $\mathbb{R}$, is $A$
   (a) open
   **Answer:** $A$ is not open. Take $0 \in A$. There is no open interval about 0 that remains entirely inside $A$.
   (b) closed
   **Answer:** To see if $A$ is closed, we need to see if $\mathbb{R} \setminus A$ is open. Now $\mathbb{R} \setminus A = (-\infty, 0) \cup \{2\} \cup [3, \infty)$. This is not an open set. Take, for example, $2 \in \mathbb{R} \setminus A$. There is no open interval about 0 that remains entirely inside $\mathbb{R} \setminus A$. As the complement of $A$ is not open, $A$ is not closed.
   (c) open and closed
   **Answer:** No.
   (d) neither open nor closed?
   **Answer:** Yes.

   Explain your answers!

3. Given $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = 2x^2y^4 + 7x^3y$ and $g : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $g(s, t) = (g_1(s, t), g_2(s, t))$ where $g_1(s, t) = s \cos t$ and $g_2(s, t) = \sin(t^2) + 2s$. Define $F : \mathbb{R}^2 \to \mathbb{R}^2$ by $F(s, t) = f(g(s, t))$.
   (a) Find the Jacobian matrix of $Df$
   **Answer:**
   $$Df = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 4xy^4 + 21x^2y & 8x^2y^3 + 7x^3 \end{bmatrix}$$

(b) FInd the Jacobian matrix of $Dg$

**Answer:**

$$Dg = \begin{bmatrix} \frac{\partial g_1}{\partial s} & \frac{\partial g_1}{\partial t} \\ \frac{\partial g_1}{\partial s} & \frac{\partial g_1}{\partial t} \end{bmatrix} = \begin{bmatrix} \cos t & -s\sin t \\ 2 & 2t\cos(t^2) \end{bmatrix}$$

(c) Using the chain rule we know $DF(s,t) = Df(g_1(s,t), g_2(s,t))Dg(s,t)$. Using the matrix multiplication find

i. $\frac{\partial F}{\partial s}$

ii. $\frac{\partial F}{\partial t}$

**Answer:**

$$\begin{aligned} DF &= \begin{bmatrix} \frac{\partial F}{\partial s} & \frac{\partial F}{\partial t} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial g_1}{\partial s} & \frac{\partial g_1}{\partial t} \\ \frac{\partial g_1}{\partial s} & \frac{\partial g_1}{\partial t} \end{bmatrix} \\ &= \begin{bmatrix} 4xy^4 + 21x^2y & 8x^2y^3 + 7x^3 \end{bmatrix} \begin{bmatrix} \cos t & -s\sin t \\ 2 & 2t\cos(t^2) \end{bmatrix} \end{aligned}$$

Hence $\frac{\partial F}{\partial s} = \cos t(4xy^4 + 21x^2y) + 2(8x^2y^3 + 7x^3)$ and
$\frac{\partial F}{\partial t} = -s\sin t(4xy^4 + 21x^2y) + 2t\cos(t^2)(8x^2y^3 + 7x^3)$.