# Math 225 Homework 3

**Due 4pm on Fri Feb 13 or Mon Feb 16, 2009.**

When writing up your solutions, pay attention to what you write. I'm interested in seeing proofs written rigorously. What does this mean? Good proofs are:

- Correct — ideally, every statement should follow from axioms or from what has been proved before.
- Concise — a proof should not contain anything that is not necessary.
- Readable — Human beings both write and read proofs. Don't be afraid to explain in words what you are doing. For example, before embarking on a long computation, it is a good idea to explain what you are doing and why you are doing it.

## 1 Problems

1. Give an example of a function $\mathbb{R}^2 \to \mathbb{R}$ which is not continuous at a point $(a, b)$ in its domain. (You must prove that it is not continuous at the point in question.)
2. Problem 1.23 page 13 Spivak.
3. Problem 1.24 page 13 Spivak.
4. Problem 1.10 page 5 Spivak.
5. Problem 1.25 page 13 Spivak.

## 2 More problems

These problems will help you remember some things from Calculus. We'll use these ideas very soon.

**Definition 1.** Given 1-variable function $f : \mathbb{R} \to \mathbb{R}$, we say that $f$ is *differentiable at $x = a$* if $\lim_{x\to a} \frac{f(x) - f(a)}{x - a}$ exists. In this case the limit is denoted by $f'(a)$.

1. Find the derivative of $f(x) = 3x - 1$ at $x = a$ using the definition. (Of course your answer will be $f'(a) = 3$, I'd like to see you using the definition.)
2. Find the derivative of $f(x) = x^2$ at $x = a$ using the definition. (Of course your answer will be $f'(a) = 2a$, I'd like to see you using the definition.)
3. Find the equation of the tangent line to $f(x) = x^2$ at the following points.
   (a) $x = -2$
   (b) $x = 0$
   (c) $x = 1$
4. Find the equation of the tangent plane to the surface $f(x, y) = x^2 + y^2$ at the following points.
   (a) $(x, y) = (0, 1)$
   (b) $(x, y) = (-1, -1)$
   (c) $(x, y) = (2, 1)$

## 3 Reminder — find my office and have a brief chat

This task is worth one quarter of your grade for HW 1. As I have been ill and not around very much, you have until February 28 to complete it.

This means you'll know where my office is when you have questions. I'll touch base with all of you, learning the names of students I haven't already met and catching up with those of you I've taught previously.