Math 225 Homework 1

Due Friday January 31, 2009.

When writing up your solutions, pay attention to what you write. I'm interested in seeing proofs written rigorously. What does this mean? Good proofs are:

- Correct — ideally, every statement should follow from axioms or from what has been proved before.
- Concise — a proof should not contain anything that is not necessary.
- Readable — Human beings both write and read proofs. Don't be afraid to explain in words what you are doing. For example, before embarking on a long computation, it is a good idea to explain what you are doing and why you are doing it.

1 Problems

These problems are worth half of your HW grade for this assignment.

1. Problem 1.1 page 4 Spivak. (Hint: Try this for $n = 1$, $n = 2$. Can you now see how to do it for any n?)
2. Problem 1.2 page 4 Spivak.
3. Problem 1.3 page 4 Spivak. (Hint: use the properties of norm found in Theorem 1-1.)
4. Problem 1.5 page 4 Spivak. (Hint: use the properties of norm found in Theorem 1-1.)
5. A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ is called **norm preserving** if $||T(x)|| = ||x||$ and **inner-product preserving** if $\langle T(x), T(y)\rangle = \langle x, y\rangle$. (Here we assume that x, y are vectors in $\mathbb{R}^n$.)
   (a) Assume that T is norm preserving and show it must be inner-product preserving as well. (Hint: Look at the properties of inner-product in Theorem 1-2.)
   (b) Assume that T is inner-product preserving and show it must be norm preserving as well.
   (c) Prove that such a linear transformation T is one-to-one. (Hint: assume it is not and try and get a contradiction.) Is T^{-1} one-to-one as well?

Remark: You may have seen these transformations referred to as **orthogonal transformations**.

6. If $x, y \in \mathbb{R}^n$ are non-zero, the **angle** between x and y is defined by

$$\angle(x, y) = \arccos\left(\frac{\langle x, y\rangle}{||x||||y||}\right).$$

The linear transformation T is **angle preserving** if

- T is one-to-one.
- For $x, y \neq 0$, we have $\angle(T(x), T(y)) = \angle(x, y)$.

(a) Prove that if T is is norm preserving, then T is angle preserving. (Hint: you may want to refer to results proved previously on this HW.)

(b) If there is a basis $x_1, \dots x_2$ of $\mathbb{R}^n$ and numbers $\lambda_1, \dots \lambda_n$ such that $T(x_i) = \lambda_i$, prove that T is angle preserving if and only if all the $|\lambda_i|$ are equal.

(c) What are all angle preserving transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$?

2 Resources

This task is worth one quarter of your HW grade for this assignment.

Throughout the semester, you may find yourself needing to look up results from linear algebra, calculus 1, calculus 2, multivariable calculus or advanced calculus. Go to the Young Science library, look at the library catalogue, and locate these texts on the shelves. (Note do not borrow these texts, or else your classmates will not be able to complete this assignment!) Also look online to see what electronic resources are available.

Write a list of (1) library and (2) online resources for Linear algebra
Write a list of (1) library and (2) online resources for Calculus (1,2, multi and advanced)

So I'll see at least four resources listed on this part of the assignment. (Do not list Wikipedia — I'm sure you can do better than that.) When listing library texts, give title, author, publisher, date published and the library's call number. When listing online resources give the web link and a brief description.

3 Find my office and have a brief chat

This task is worth one quarter of your HW grade for this assignment. Unlike the rest of the assignment you have until February 13 to complete it.

This means you'll know where my office is when you have questions. I'll touch base with all of you, learning the names of students I haven't already met and catching up with those of you I've taught previously.