Math 225

Comments on exam problem solutions

1 Partial Derivatives and Jacobian review

1. Find the partial derivatives of $G(F(x+y), F(y+z))$. Let $H(x,y,z) = G(F(x+y), F(y+z)) = G(s,t)$, where $s = F(x+y) = F(u)$ and $t = F(y+z) = F(v)$, where $u(x,y,z) = x+y$ and $v(x,y,z) = y+z$. Now we can use the chain rule:

$$\frac{\partial H}{\partial x} = \frac{\partial G}{\partial s}\frac{\partial s}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial G}{\partial t}\frac{\partial t}{\partial v}\frac{\partial v}{\partial x} = \frac{\partial G}{\partial s}F'(u)1 + \frac{\partial G}{\partial t}F'(v)0 = \frac{\partial G}{\partial s}F'(x+y)$$

or $\frac{\partial H}{\partial x} = D_1G\,F(x+y)$

$$\frac{\partial H}{\partial y} = \frac{\partial G}{\partial s}\frac{\partial s}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial G}{\partial t}\frac{\partial t}{\partial v}\frac{\partial v}{\partial y} = \frac{\partial G}{\partial s}F'(u)1 + \frac{\partial G}{\partial t}F'(v)1 = \frac{\partial G}{\partial s}F'(x+y) + \frac{\partial G}{\partial t}F'(y+z)$$

or $\frac{\partial H}{\partial y} = D_1G\,F(x+y) + D_2G\,F'(y+z)$

$$\frac{\partial H}{\partial z} = \frac{\partial G}{\partial s}\frac{\partial s}{\partial u}\frac{\partial u}{\partial yz} + \frac{\partial G}{\partial t}\frac{\partial t}{\partial v}\frac{\partial v}{\partial z} = \frac{\partial G}{\partial s}F'(u)0 + \frac{\partial G}{\partial t}F'(v)1 = \frac{\partial G}{\partial t}F'(y+z)$$

or $\frac{\partial H}{\partial y} = D_2G\,F'(y+z)$

2. Fine

3. Fine

2 Inverse Function Theorem

1. Useful question, but not worth discussing for the second midterm. Be sure you can do it for the final!

2. Previous HW question

3. Note that the fact that f is not 1-1 everywhere does not contradict the Inverse Function Theorem. The InFT guarantees that locally, f is 1-1. That is at each point a where $\det DF(a) \neq 0$ there is an open set $a \in V$ and an open set $f(a) \in W$ such that $f : V \to W$ is 1-1.

3 Implicit Function Theorem

1. Fine

2. There is a problem with the question. The solutions are also completely incorrect. We have $f : \mathbb{R}^3 \to \mathbb{R}$, a function that is continuously differentiable everywhere. Now $M = \begin{bmatrix} -\cos y \end{bmatrix}$ since $DF = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{bmatrix} = \begin{bmatrix} y + e^x & x + e^x + z \sin y & -\cos y \end{bmatrix}$. Hence $\det(M) = -\cos y$

   Given an (a, b, c) such that $f(a, b, c) = 0$, we can apply the Implicit Function Theorem provided $\det M \neq 0$. That is provided that $b \neq n\pi/2$ (for $n = \pm 1, \pm 2, \pm 3, \dots$). Given such an (a, b, c), then ImFT tells us that there is an open set $V \subset \mathbb{R}^2$ with $(a, b) \in V$ and an open set $W \subset \mathbb{R}$ with $c \in W$ such that for all $(x, y) \in V$, there is a unique $g(x, y) \in W$ such that $f(x, y, g(x, y)) = 0$. The function g is differentiable.

3. Fine.

4 Integration

1. Fine

2. Solution is OK, but note some details are missing in the computation of $L(f, P)$ and $U(f, P)$. (Also note that the integral should be over $[0, 2] \times [0, 1]$ not $[0, 1] \times [0, 1]$.)

3. Don't worry about this one for midterm 2, but do think about it.

4. Fine.