Inverse Function Theorem

1) Spivak pg. 39, # 236

Let $A \subset \mathbb{R}^n$ be an open set and $f: A \to \mathbb{R}^n$ a continuously differentiable 1-1 function s.t. $\det f'(x) \neq 0$ $\forall x$. Show that $f(A)$ is an open set and $f^{-1}: f(A) \to A$ is differentiable. Show that $f(B)$ is open for any open set $B \subset A$.

2) Spivak pg. 40 # 2-39.

Use the function $f: \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} x/2 + x^2 \sin 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

to show that continuity of ~~and~~ the derivative cannot be eliminated from the hypothesis of the Inverse Function Theorem.

3) Spivak pg 40. #2 - 38 (b)

Define $f: \mathbb{R}^2 \to \mathbb{R}^2$ by $f(x,y) = (e^x \cos y, e^x \sin y)$.
Show that $\det f'(x,y) \neq 0$ $\forall (x,y)$, but f is not 1-1.