HW 3 Solutions Math 211

3.1

1) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \operatorname{im} A = \operatorname{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right)$ a plane in $\mathbb{R}^3$ through origin (xy-plane)

$\ker A = \operatorname{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)$ x-axis (line through origin)

$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \operatorname{im} A = \operatorname{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)$ x-axis line through origin.

$\ker A = \operatorname{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right)$ xy-plane plane through origin in $\mathbb{R}^3$

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ s \\ 0 \end{bmatrix}$$

$$\vec{x} = t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \operatorname{Im} A = \{\vec{0}\}$, $\ker A = \mathbb{R}^3$

2) $A = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ or $\begin{bmatrix} 2 & 0 \\ 3 & 0 \end{bmatrix}$ or $\begin{bmatrix} 2 & 4 & 0 \\ 3 & 6 & 0 \end{bmatrix}$ (columns must be $k \begin{bmatrix} 2 \\ 3 \end{bmatrix}$)

$\operatorname{Im} A = \operatorname{span}\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix} \right)$ $\quad$ many such matrices A

3) B any $n \times 3$ matrix s.t. $B \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \vec{0}$ $\quad$ require $1\vec{v}_1 - 2\vec{v}_2 + 3\vec{v}_3 = \vec{0}$ for $[\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3] = B$

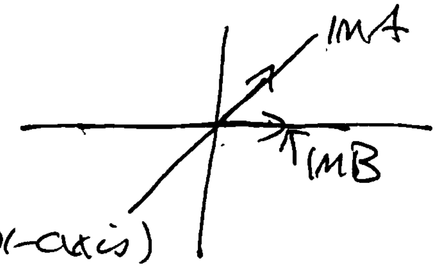
ex. $B = \begin{bmatrix} 1 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$ $\quad$ many such matrices B.

4) $A \quad B = \operatorname{rref}(A)$

a) $\ker A = \{\vec{x} \mid A\vec{x} = \vec{0}\} = \{\vec{x} \mid (\operatorname{rref}(A))\vec{x} = \vec{0}\} = \{\vec{x} \mid B\vec{x} = \vec{0}\}$
$= \ker B$.

b) $\operatorname{Im} A \neq \operatorname{Im} B$ in general

ex $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$



$\operatorname{Im} A = \operatorname{span}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\operatorname{im} B = \operatorname{span}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (x-axis)

($y = x$)

$\boxed{\text{Section 3.2}}$

1a) W is <u>not</u> a subspace $\vec{0} \notin W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + y + z = 1 \right\}$

$0 + 0 + 0 \neq 1$

b) W is <u>not</u> a subspace $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x \leq y \leq z \right\}$

If $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in W$ then $-1\begin{bmatrix} x \\ y \\ z \end{bmatrix} \notin W$

since $-x \geq -y \geq -z$.

② Subspaces of $\mathbb{R}^3$ are $\{\vec{0}\}$, lines through $\vec{0}$, planes thru' $\vec{0}$ & $\mathbb{R}^3$. Let W be a subspace of $\mathbb{R}^3$.

If $W \neq \vec{0}$ then there is a $\vec{v} \in W$ & $\vec{v} \neq \vec{0}$. So $W = \operatorname{span}(\vec{v})$ - This is a line through $\vec{0}$. Suppose W is not this, then $\exists$ there is a $\vec{u} \in W$ & $\vec{u} \neq k\vec{v}$

Hence $W = \operatorname{span}(\vec{v}, \vec{u})$ which is a plane through $\vec{0}$.

Suppose W is none of these! Then there is a $\vec{w} \in W$ & $\vec{w} \neq k\vec{u} + l\vec{v}$ $k, l \in \mathbb{R}$. Then $W = \operatorname{span}(\vec{u}, \vec{v}, \vec{w}) = \mathbb{R}^3$.

③ a) $\begin{bmatrix} -6 \\ -9 \end{bmatrix}$ is redundant since $\begin{bmatrix} -6 \\ -9 \end{bmatrix} = -3\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

b) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is redundant since $0\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (non-trivial relation)

OR $0\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

(c) $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ is redundant $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \neq k\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $\begin{bmatrix} 3 \\ 4 \end{bmatrix} = -\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$c_1\begin{bmatrix}1\\2\end{bmatrix}+c_2\begin{bmatrix}2\\3\end{bmatrix}=\begin{bmatrix}3\\4\end{bmatrix}$ solve for c_1 & c_2 $\begin{bmatrix}1&2\\2&3\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix}=\begin{bmatrix}3\\4\end{bmatrix}$

$$\left[\begin{array}{cc|c}1&2&3\\2&3&4\end{array}\right]\begin{array}{l}\\-2\mathrm{I}\end{array}\rightarrow\left[\begin{array}{cc|c}1&2&3\\0&-1&-2\end{array}\right]\rightarrow\left[\begin{array}{cc|c}1&2&3\\0&1&2\end{array}\right]\begin{array}{l}-2\mathrm{II}\\ \\ \end{array}\rightarrow\left[\begin{array}{cc|c}1&0&-1\\0&1&2\end{array}\right]$$

d) $\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}1\\2\\0\end{bmatrix},\begin{bmatrix}1\\2\\3\end{bmatrix}$ linearly independent

no vectors are redundant (use the zeros).

4) a) $\operatorname{im}\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}=\operatorname{span}\left(\begin{bmatrix}1\\4\end{bmatrix},\begin{bmatrix}2\\5\end{bmatrix},\begin{bmatrix}3\\6\end{bmatrix}\right)$

$\begin{bmatrix}1\\4\end{bmatrix},\begin{bmatrix}2\\5\end{bmatrix}$ linearly indep. & a basis. ($\begin{bmatrix}3\\6\end{bmatrix}$ redundant)

$$\left(\left[\begin{array}{cc|c}1&2&3\\4&5&6\end{array}\right]\begin{array}{l}\\-4\mathrm{I}\end{array}\rightarrow\left[\begin{array}{cc|c}1&2&3\\0&-3&-6\end{array}\right]\rightarrow\left[\begin{array}{cc|c}1&2&3\\0&1&2\end{array}\right]\rightarrow\left[\begin{array}{cc|c}1&0&-1\\0&1&2\end{array}\right]\right)$$

b) basis is linearly independent column vectors.

Im A has basis $\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$

$\boxed{\text{Note}}$ $\begin{bmatrix}0\\0\\0\end{bmatrix},\begin{bmatrix}2\\0\\0\end{bmatrix},\begin{bmatrix}3\\4\\5\end{bmatrix}$ redundant

5) a) $A=\begin{bmatrix}2&2\\1&1\end{bmatrix}$ $\vec{v}_1=\vec{v}_2\Rightarrow\vec{0}=\vec{v}_1-\vec{v}_2$ $\begin{bmatrix}1\\-1\end{bmatrix}\in\ker A$

b) $\begin{bmatrix}1&0&2\\0&1&3\\0&0&0\end{bmatrix}$ $\vec{v}_3=2\vec{v}_1+3\vec{v}_2$

$\Rightarrow\vec{0}=2\vec{v}_1+3\vec{v}_2-\vec{v}_3$ $\begin{bmatrix}2\\3\\-1\end{bmatrix}\in\ker A.$

HW7 T/F Solutions

① <u>False</u> $\quad {\scriptstyle 3}\underbrace{\begin{bmatrix} \quad A \quad \end{bmatrix}}_{4}$ $\quad$ 3 rows $\Rightarrow$ 4 colums $\quad$ Image is a subspace of $\mathbb{R}^3$ (not $\mathbb{R}^4$).

② <u>True</u> $\quad$ It's the definition

③ <u>True</u> $\quad 2\vec{u} + 3\vec{v} + 4\vec{w} = 5\vec{u} + 6\vec{v} + 7\vec{w}$

$\Rightarrow 3\vec{u} + 3\vec{v} + 3\vec{w} = \vec{0}$ $\quad$ nontrivial relation

so vectors are dependent.

④ <u>True</u> $\quad$ definition of a subspace

$\vec{u}, \vec{v}, \vec{w} \in V \subseteq \mathbb{R}^n \qquad 2\vec{u} - 3\vec{v} + 4\vec{w} \in V$

⑤ <u>False</u> $\quad$ ex $\mathbb{R}^2$ $\quad$ pick $\vec{v} \neq \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & $\vec{v} \neq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$V = \text{span}(\vec{v})$ a subspace of $\mathbb{R}^2$ - line through origin.

& $V \neq \{\vec{0}\}$

⑥ <u>True</u> $\quad \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ linearly independent

none are redundant - true for $\vec{v}_1, \vec{v}_2$ & $\vec{v}_3$ too.