Homework 7 Math 211

Due 4pm Friday October 23, 2009.

Section 3.1

1. For the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, describe the images and kernels of the matrices A, A^2 and A^3 geometrically. (Hint: you'll first need to compute A^2 and A^3.)

2. Give an example of a matrix A such that $\text{im}(A)$ is spanned by the vector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

3. Give an example of a matrix B such that $\ker(B)$ is spanned by the vector $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$.

4. Consider a matrix A and let $B = \text{rref}(A)$.

   (a) Is $\ker(A)$ necessarily equal to $\ker(B)$? Explain.

   (b) Is $\text{im}(A)$ necessarily equal to $\text{im}(B)$? Explain.

Section 3.2

1. Which of the sets W are subspaces of $\mathbb{R}^3$? You must justify your answer

   (a) $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + y + z = 1 \right\}$

   (b) $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x \leq y \leq z \right\}$

2. Give a geometrical description of all subspaces of $\mathbb{R}^3$. You must justify your answer. (Hint: follow the argument given in example 2 but for the new setting.)

3. In the following exercise, determine which vectors are redundant. (Use pencil and paper, but check on computer.) Thus determine whether the given vectors are linearly independent.

   (a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -6 \\ -9 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

4. Find a basis of the image of the following matrices

   (a) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

5. Find a redundant column vector in the given matrix and write it as a linear combination of the preceeding columns. Use this to write a nontrivial relation among the columns and so find a nonzero vector in the kernel of A.

   (a) $A = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

True/False

Please submit the answers to these questions on a separate page with your name on it.

Are the following statements True or False? You must give a reason for your answer to receive full credit.

1. The image of a 3×4 matrix is a subspace of $\mathbb{R}^4$.
2. The span of vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ consists of all linear combinations of vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$
3. If $2\vec{u} + 3\vec{v} + 4\vec{w} = 5\vec{u} + 6\vec{v} + 7\vec{w}$, then vectors $\vec{v}, \vec{u}, \vec{w}$ are linearly dependent.
4. If vectors $\vec{v}, \vec{u}, \vec{w}$ are in a subspace V of $\mathbb{R}^n$, then vector $2\vec{u} - 3\vec{v} + 4\vec{w}$ must be in V as well.
5. If a subspace V of $\mathbb{R}^n$ contains none of the standard vectors $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n$, then V consists of the zero vector only.
6. If vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ are linearly independent, then vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ must be linearly independent as well.

Optional Reading

Linear algebra has applications throughout mathematics, the sciences and social sciences. You now know enough to start applying your knowledge to your areas of interest. For example:

1. If you are interested in **Coding Theory** read exercises 53 and 54 on page 112. It's about Hamming codes, which are an example of error correcting codes.