

# Homework 6 Math 211

**Due 4pm Monday October 19, 2009.**

## Section 2.4

1. If  $A$  is an invertible matrix and  $c$  is a nonzero scalar, is the matrix  $cA$  invertible? If so, what is the relationship between  $A^{-1}$  and  $(cA)^{-1}$ .
2. For two invertible  $n \times n$  matrices  $A$  and  $B$ , determine which of the following formulas are necessarily true. Please give a brief explanation why or why not.
  - (a)  $(A + B)^2 = A^2 + 2AB + B^2$
  - (b)  $A^2$  is invertible and  $(A^2)^{-1} = (A^{-1})^2$
  - (c)  $(A + B)$  is invertible and  $(A + B)^{-1} = A^{-1} + B^{-1}$
  - (d)  $(A - B)(A + B) = A^2 - B^2$
  - (e)  $ABB^{-1}A^{-1} = I_n$ .
  - (f)  $ABA^{-1} = B$
  - (g)  $(ABA^{-1})^3 = AB^3A^{-1}$
  - (h)  $(I_n + A)(I_n + A^{-1}) = 2I_n + A + A^{-1}$
  - (i)  $A^{-1}B$  is invertible and  $(A^{-1}B)^{-1} = B^{-1}A$

## Section 3.1

1. Describe the images and kernels of the following transformations geometrically.
  - (a) Reflection about the line  $y = 2x$  in  $\mathbb{R}^2$ .
  - (b) Orthogonal projection onto the plane  $x + 2y + 3z = 0$  in  $\mathbb{R}^3$ .
  - (c) Rotation through  $\pi/3$  counterclockwise in  $\mathbb{R}^2$ .
2. For each matrix  $A$  below, find the vectors that span the kernel of  $A$ . Use paper and pencil, but it is OK to check your answer on the computer.

- (a)  $\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$
- (b)  $[1 \ 2 \ 3]$
- (c)  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
- (d)  $\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$

3. For each matrix  $A$  below, find the vectors that span the image of  $A$ . Give as few vectors as possible. Use paper and pencil, but it is OK to check your answer on the computer.

- (a)  $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$
- (b)  $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$
- (c)  $\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$

### True/False

**Please submit the answers to these questions on a separate page with your name on it.**

Are the following statements True or False? You must give a reason for your answer to receive full credit.

1. There is an invertible  $2 \times 2$  matrix  $A$  such that  $A^{-1} = -A$ .
2. If matrix  $A$  commutes with  $B$  and  $B$  commutes with  $C$ , then matrix  $A$  must commute with  $C$ .
3. There exists an invertible  $10 \times 10$  matrix that has 92 ones among its entries.
4. If matrices  $A$  and  $B$  are both invertible, then  $A + B$  must be invertible as well.
5. If  $A^2$  is invertible, then matrix  $A$  must be invertible as well.

### Optional Reading

Linear algebra has applications throughout mathematics, the sciences and social sciences. You now know enough to start applying your knowledge to your areas of interest. For example:

1. If you are interested in **Coding Theory** read exercises 53 and 54 on page 112. It's about Hamming codes, which are an example of error correcting codes.