HW5 True/False Solutions

1. $T\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}y\\1\end{bmatrix}$ is not linear (FALSE)

$$T\left(k\begin{bmatrix}x\\y\end{bmatrix}\right) = T\left(\begin{bmatrix}kx\\ky\end{bmatrix}\right) = \begin{bmatrix}ky\\1\end{bmatrix} \neq \begin{bmatrix}ky\\k\end{bmatrix} = k\begin{bmatrix}y\\1\end{bmatrix} = kT\left(\begin{bmatrix}x\\y\end{bmatrix}\right)$$

2. A 2×2 upper triangular (TRUE)

$$A^2 = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} \qquad A = \begin{bmatrix}a & b\\0 & d\end{bmatrix} \qquad A^2 = \begin{bmatrix}a & b\\0 & d\end{bmatrix}\begin{bmatrix}a & b\\0 & d\end{bmatrix}$$

$$A^2 = \begin{bmatrix}a^2 & ab+bd\\0 & d^2\end{bmatrix}$$

Solve $a^2=1$ $a=\pm 1$ $d^2=1$ $d=\pm 1$

$ab+bd=1$

$a=1$	$d=1$	$2b=1$	$b=\frac{1}{2}$	$A=\begin{bmatrix}1 & \frac{1}{2}\\0 & 1\end{bmatrix}$
$a=-1$	$d=-1$	$-2b=1$	$b=-\frac{1}{2}$	$A=\begin{bmatrix}-1 & -\frac{1}{2}\\0 & -1\end{bmatrix}$
$a=1$	$d=-1$	$b-b=0\neq 1$	no solution	
$a=-1$	$d=1$	$-b+b=0\neq 1$	no solution.	

3. $\begin{bmatrix}k & -2\\5 & k-6\end{bmatrix}$ invertible for all real k (TRUE)

matrix is invertible $\Leftrightarrow$ $k(k-6)+10\neq 0$

$= k^2-6k+10\neq 0 \Rightarrow k\neq \frac{6\pm\sqrt{36-40}}{2}$

But $\sqrt{-4}$ not real so $k^2-6k+10\neq 0$ for all real k!

4. 2×3 A 3×2 B (TRUE)

$AB = I_2$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\\0 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \qquad \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

5. 3×2 A 2×3 B $AB = I_3$ FALSE

If $AB = I_3$ Then $AB\vec{x} = I_3\vec{x} = \vec{x}$

So $AB\vec{x} = \vec{y}$ has a unique solution for each $\vec{y}$!

But Pick $\vec{y} = 0$ $AB\vec{x} = \vec{0}$

Consider $B\vec{x} = \vec{0}$ B is 2×3, more variables than equations

$B\vec{x} = \vec{0}$ has ∞ # solutions ($\vec{x} = \vec{0}$ solution.)

So $AB\vec{x} = \vec{0}$ has ∞ # solutions.

So $AB = I_3$ Not true.

Long solution involves solving non linear equations.

Yuk.