Homework 5 Math 211

Due 4pm October 9, 2009.

Section 2.3

1. Consider an $n \times p$ matrix A, a $p \times m$ matrix B, and a scalar k. Show that $(kA)B = A(kB) = k(AB)$. (Hint: make sure you do this for these arbitrary matrices and not a particular example. To help you with notation, take a look at Theorem 2.3.4 on pg. 73.)

2. Consider matrix $D_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$. We know that the linear transformation $T(\vec{x}) = D_\alpha(\vec{x})$ is a counterclockwise rotation through angle α.

   (a) For two angles α and β, consider the products $D_\alpha D_\beta$ and $D_\beta D_\alpha$. Arguing geometrically, describe the linear transformations $\vec{y} = D_\alpha D_\beta \vec{x}$ and $\vec{y} = D_\beta D_\alpha \vec{x}$. Are the two transformations the same?

   (b) Compute the products $D_\alpha D_\beta$ and $D_\beta D_\alpha$. Do the results makes sense in terms of your answer in part (a)? (Hint: recall the trigonometric identities $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$, $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$.)

3. In the following exercises, find a 2×2 matrix A that has the given properties. (Hint: it helps to think of a geometrical example.)

   (a) $A \neq I_2$, $A^2 = I_2$.

   (b) $A^2 \neq I_2$, $A^3 = I_2$.

   (c) $A \neq I_2$, $A^2 = A$.

   (d) $A \neq 0$, $A^2 = 0$.

Section 2.4

1. Which of the following linear transformations $T : \mathbb{R}^3 \to \mathbb{R}^3$ are invertible? Describe the inverse if it exists. (Don't write down matrices, just give a verbal description.)

   (a) Reflection about a plane

   (b) Orthogonal projection onto a plane

   (c) Scaling by a factor of 5 (that is, $T(\vec{v}) = 5\vec{v}$ for all $\vec{v} \in \mathbb{R}^3$)

   (d) Rotation about an axis

2. Decide whether the following matrices are invertible. If they are, find the inverse. Do the computations with paper and pencil and show some work. Remember, it is a good idea to check your answers on the computer! (Hint: you can use any theorems you know to help you decide whether a matrix is invertible or not. Just be sure to write down the reason for your answer!)

   (a) $\begin{bmatrix} 3 & 3 \\ 5 & 8 \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$

   (e) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

   (f) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 7 & 11 \\ 3 & 7 & 14 & 25 \\ 4 & 11 & 25 & 50 \end{bmatrix}$

3. Find all matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $ad - bc = 1$ and $A^{-1} = A$.

True/False

Please submit the answers to these questions on a separate page with your name on it.

Are the following statements True or False? You must give a reason for your answer to receive full credit.

1. The function $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ 1 \end{bmatrix}$ is a linear transformation.

2. There exists an upper triangular 2×2 matrix A such that $A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

3. Matrix $\begin{bmatrix} k & -2 \\ 5 & k-6 \end{bmatrix}$ is invertible for all real numbers k.

4. There exists a 2×3 matrix A and a 3×2 matrix B such that $AB = I_2$.

5. There exists a 3×2 matrix A and a 2×3 matrix B such that $AB = I_3$.

Optional Reading

Linear algebra has applications throughout mathematics, the sciences and social sciences. You now know enough to start applying your knowledge to your areas of interest. For example:

1. If you are interested in **Economics** read exercises 49 and 50 on pages 90, 91. You'll need to also read some exercises in Chapter 1 to get a complete understanding. Follow up by reading exercises 101, 102 and 103.

2. If you are interestedin **Biology** and genetics, take a look at the simple model in exercise 105 page 95.

3. If you are interested in **Physics** or **Engineering** take a look at the series of exercises 45, 104, 106, 107, 108 on optics on pages 95-98.