

# Solutions MW 4 T/E

① (F)  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$  upper  $\Delta$  matrix  
rank 2  $\neq$  1  $\leftarrow$  # 1's on diagonal

②  $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x-y \\ y-x \end{bmatrix}$  is a linear transf.

True T has matrix  $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

③ False  $AB \neq BA$  ex  $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ -15 & 5 \end{bmatrix}$   
 $\begin{bmatrix} 0 & 1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} \neq \begin{bmatrix} -5 & 3 \\ -15 & 5 \end{bmatrix}$

④ True  $\mathbb{R}^5 \xrightarrow{B} \mathbb{R}^4 \xrightarrow{A} \mathbb{R}^3$   
AB A 3x4  
B 4x5  
AB 3x5

⑤ True  $\begin{bmatrix} 5 & 6 \\ -6 & 5 \end{bmatrix}$  is a rotation and a scaling  $\Rightarrow 5^2 + 6^2 = r^2$   
 $\Rightarrow r = \sqrt{61}$   
 $\begin{bmatrix} r \cos \theta & -r \sin \theta \\ r \sin \theta & r \cos \theta \end{bmatrix}$

⑥ FALSE  $\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}$  Does not represent a rotation  $\frac{1}{2}^2 + \frac{1}{2}^2 = \frac{1}{2} \neq 1$   
 $\cos^2 \theta + \sin^2 \theta = 1$ .

⑦ A 4x3 rank A = 3  $\text{ref } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & a \end{bmatrix}$  Assume  $A\vec{v} = A\vec{w}$ .

TRUE  
 $A\vec{v} - A\vec{w} = \vec{0}$   $A\vec{v} - A\vec{w} = A(\vec{v} - \vec{w}) = \vec{0}$  But  $A\vec{x} = \vec{0}$  has a unique solution.

So  $\vec{v} - \vec{w} = \vec{0} \Rightarrow \vec{v} = \vec{w}$

⑧  $A\vec{x} = \vec{b}$  consistent  $\Leftrightarrow \text{rank}(A) = \text{rank}(A|\vec{b})$   
consistent  $\Leftrightarrow \text{ref}[A|\vec{b}]$  has no row  $[0 \dots 0 | 1]$

TRUE

⑨ TRUE  $A\vec{x} = \vec{0}$   $B\vec{x} = \vec{0}$  have same solutions  
 $\text{ref}(A)$  must match  $\text{ref}(B)$

Do a case by case analysis  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} a \\ t \end{bmatrix}$   $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$   $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ t \end{bmatrix}$   
 $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} t \\ 0 \end{bmatrix}$   $\begin{bmatrix} 1 & a \\ 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} -a \\ t \end{bmatrix}$   $\text{ref}(A) \neq \text{ref}(B) \Rightarrow$  different solutions.

