Homework 4 Math 211

Due 4pm October 2, 2009.

Section 2.2

1. The matrix $\begin{bmatrix} -0.8 & -0.6 \\ 0.6 & -0.8 \end{bmatrix}$ represents a rotation. Find the angle of rotation in radians.

2. Find the scaling matrix A that transforms $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ into $\begin{bmatrix} 8 \\ -4 \end{bmatrix}$.

3. Find the matrix B that transforms $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

4. Find the rotation matrix C that transforms $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ into $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

5. Find the shear matrix D that transforms $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} 7 \\ 3 \end{bmatrix}$.

6. Interpret the following linear transformation geometrically: $T(\vec{x}) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \vec{x}$.

Section 2.1

1. Find the inverse of the linear transformation $\begin{bmatrix} y_1 = x_1 + 7x_2 \\ y_2 = 3x_1 + 20x_2 \end{bmatrix}$.

2. Decide whether the following matrices are invertible and find the inverse if it exists.

   (a) $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$

3. Prove the following facts:

   (a) The 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible if and only if $ad - bc \neq 0$. (Hint: consider the cases $a \neq 0$ and $a = 0$ separately.)

   (b) If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible, then

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

   Note: it is worth memorizing this formula!

4. For which values of k is the matrix $\begin{bmatrix} 2 & 3 \\ 5 & k \end{bmatrix}$ invertible?

Section 2.3

1. If possible, compute the following matrix products. (Do it with pencil and paper, but feel free to check your answer on the computer.)

   (a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

   (d) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

   (e) $\begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 3 & 1 \\ 1 & 1 \end{bmatrix}$

   (f) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$

   (g) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

(h) $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

True/False

Please submit the answers to these questions on a separate page with your name on it.
Are the following statements True or False? You must give a reason for your answer to receive full credit.

1. The rank of any upper triangular matrix is the number of nonzero entries on its diagonal.
2. The function $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x - y \\ y - x \end{bmatrix}$ is a linear transformation.
3. The formula $AB = BA$ holds for all $n \times n$ matrices A and B.
4. If A is a 3×4 matrix and B is a 4×5 matrix, then AB will be a 5×3 matrix.
5. The matrix $\begin{bmatrix} 5 & 6 \\ -6 & 5 \end{bmatrix}$ represents a rotation combined with a scaling.
6. Matrix $\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}$ represents a rotation.
7. If A is a 4×3 matrix of rank 3 and $A\vec{v} = A\vec{w}$ for two vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^3$, then $\vec{v}$ and $\vec{w}$ must be equal.
8. The linear system $A\vec{x} = \vec{b}$ is consistent if (and only if) $\text{rank}(A) = \text{rank}\,[\,A \,|\, \vec{b}\,]$.
9. If A and B are two 2×2 matrices such that the equations $A\vec{x} = \vec{0}$ and $B\vec{x} = \vec{0}$ have the same solutions, then $\text{rref}(A)$ must be equal to $\text{rref}(B)$.