

Math 211 Solutions T/F HW 2

1. (T) Assume that $\text{rref}(A) = \text{rref}(B)$.

Solutions of $A\vec{x} = \vec{0}$ found via $\text{rref}(A|\vec{0})$

Solutions of $B\vec{x} = \vec{0}$ - - - $\text{rref}(B|\vec{0})$

By assumption $\text{rref}(A|\vec{0}) = \text{rref}(B|\vec{0})$

$\therefore A\vec{x} = \vec{0}$ & $B\vec{x} = \vec{0}$ have the same solutions.

2. (F) 5×5 matrix of rank 4.

If $A\vec{x} = \vec{0}$ has only solution $\vec{x} = \vec{0}$, this means the solution is unique so $\text{rref}(A) = I_5$ & $\text{rank}(A) = 5 \neq 4$.

OR $A\vec{x} = \vec{0} \rightarrow$ consider $\text{rref}(A|\vec{0})$ This has rank 4 too.

$\text{rref}(A|\vec{0})$ has a row $[0\ 0\ 0\ 0\ 0\ |\ 0]$ ~~is~~ has one free variable with infinite solutions. (Case that

$A\vec{x} = \vec{0}$ is inconsistent can't occur by assumption.)

5. (F) Note that $A(k\vec{x}) = kA\vec{x}$ for all scalars k

$$A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad A \begin{bmatrix} 2 \\ 2 \end{bmatrix} = A(2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}) = 2A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \neq \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

