Homework 2 Math 211

Due 4pm September 21, 2009.

Section 1.3

1. Find the rank of the following matrices

   (a) $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 1 & -1 \\ 3 & -5 & 13 \\ 1 & -2 & 5 \end{bmatrix}$

2. Consider a linear system of three equations with three unknowns. We are told that the system has a unique solution. What does the reduced row-echelon form of the coefficient matrix look like? Explain your answer.

3. Consider a linear system of three equations with two unknowns. We are told that the system has a unique solution. What does the reduced row-echelon form of the coefficient matrix look like? Explain your answer.

4. Let A be a 4×4 matrix and let $\vec{b}$ and $\vec{c}$ be two vectors in $\mathbb{R}^4$. We are told that the system $A\vec{x} = \vec{b}$ is inconsistent. What can you say about the number of solutions of $A\vec{x} = \vec{c}$? Explain your answer.

5. If the rank of a 5×3 matrix is 3, what is $\text{rref}(A)$?

6. Compute the products $A\vec{x}$ in the following exercises (if the products are defined). (Make sure you can do these with paper and pencil.)

(a) $\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 1 \\ -3 & 1 & 1 \\ 1 & -3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

7. Find $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -3 & 2 \\ 0 & 1 \end{bmatrix}$.

8. Find $3 \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 2 \end{bmatrix}$.

9. (a) Write the system $\begin{bmatrix} x + 2y = 7 \\ 3x + y = 11 \end{bmatrix}$ in vector form

   (b) Use your answer to (a) to represent the system geometrically. Solve the system and represent the solution geometrically. (Hint: Read example 13.)

10. Is the vector $\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$?

11. A linear system of the form $A\vec{x} = \vec{0}$ is called *homogeneous*. Justify the following facts:

   (a) All homogeneous systems are consistent.

   (b) A homogeneous system with fewer equations than unknowns has infinitely many solutions.

   (c) If $\vec{x}_1$ and $\vec{x}_2$ are solutions of the homogeneous system $A\vec{x} = \vec{0}$, then $\vec{x}_1 + \vec{x}_2$ is as well.

   (d) If $\vec{x}$ is a solution of the homogeneous system $A\vec{x} = \vec{0}$, then $k\vec{x}$ is as well. (Here k is a constant.)

True/False

Please submit the answers to these questions on a separate page with your name on it.

Are the following statements True or False? You must give a reason for your answer to receive full credit.

1. If two matrices A and B have the same reduced row-echelon form, then the equations $A\vec{x} = \vec{0}$ and $B\vec{x} = \vec{0}$ must have the same solutions.

2. There exists a 5×5 matrix A of rank 4 such that the system $A\vec{x} = \vec{0}$ has only the solution $\vec{x} = \vec{0}$.

3. There exists a 3×4 matrix with rank 4.

4. If A and B are matrices of the same size, then the formula $\text{rank}(A + B) = \text{rank}(A) + \text{rank}(B)$ must hold.

5. There exists a 2×2 matrix A such that $A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $A \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

6. There exists a matrix A such that $A \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$.

7. Matrix $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is in reduced row echelon form.

Extra practice - do NOT submit for grading.

1. Find solutions to the following linear systems using Gauss-Jordan elimination as discussed in Section 1.2 of the text. Show all your work.

$$\begin{bmatrix} x_4 + 2x_5 - x_6 = 2 \\ x_1 + 2x_2 + x_5 - x_6 = 0 \\ x_1 + 2x_2 + 2x_3 - x_5 + x_6 = 2 \end{bmatrix}$$

2. Consider the equations $\begin{bmatrix} y + 2kz = 0 \\ x + 2y + 6z = 2 \\ kx + 2z = 1 \end{bmatrix}$, where k is an arbitrary constant.

   (a) For which values of k does this system have a unique solution?

   (b) When is there no solutions?

   (c) When are there infinitely many solutions?