

HW1 Feedback 211

Section 1.1 →

Q1-3 well done - make sure that all steps are explained in your computations!

Q4) 
$$\begin{bmatrix} x+y-z = -2 \\ 3x-5y+13z = 18 \\ x-2y+5z = k \end{bmatrix} \begin{matrix} -3I \\ -I \end{matrix} \rightarrow \begin{bmatrix} x+y-z = -2 \\ -8y+16z = 24 \\ -3y+6z = k+2 \end{bmatrix} \div -8$$

$$\rightarrow \begin{bmatrix} x+y+z = -2 \\ y-2z = -3 \\ -3y+6z = k+2 \end{bmatrix} \begin{matrix} -I \\ +3II \end{matrix} \rightarrow \begin{bmatrix} x+z = 1 \\ y-2z = -3 \\ 0 = k-7 \end{bmatrix}$$

a) The system does not have a unique solution (can never have  $z = *$ ).

When  $k=7$  last equation is  $0=0$ . So  $z$  is a free variable and there are  $\infty$  solutions

b/c) When  $k=7$

$$\begin{aligned} x &= 1 - z \\ y &= -3 + 2z \\ z &= z \end{aligned}$$

when  $k \neq 7$  Then last equation reads  $0 = k-7 \neq 0$   
 Thus the system is inconsistent  $\Rightarrow$  no solutions.

Q5) Most people lost points for not considering the cases  
 degree 0  $f(t) = c$  a constant  $\Rightarrow f'(t) = 0 \neq 1 = f'(1)$ .  
 degree 1  $f(t) = at + b$   $\Rightarrow$  not possible.

Here  $f'(t) = a$   $f'(1) = a$ . Solve

$$\begin{bmatrix} a+b=3 \\ 2a+b=6 \\ a=1 \end{bmatrix} \begin{matrix} -2I \\ -I \end{matrix} \rightarrow \begin{bmatrix} a+b=3 \\ -b=0 \\ -b=-2 \end{bmatrix} \rightarrow \begin{bmatrix} a+b=3 \\ b=0 \\ 0=2 \end{bmatrix}$$

so no solution.

## Section 1.2

$$1b) \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 5 \\ 3 & 4 & 2 \end{bmatrix} \begin{matrix} \\ -2I \\ -3I \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 3 \\ 0 & 1 & -1 \end{bmatrix} \begin{matrix} \\ \\ \div -3 \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{matrix} -II \\ \\ -II \end{matrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} x=2 \\ y=2 \end{matrix}$$

→ This equation gives no information.

$$1d) \begin{bmatrix} 0 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 4 & 8 & 0 \end{bmatrix} \begin{matrix} \\ \div 4 \end{matrix} \begin{bmatrix} 0 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{bmatrix} \begin{matrix} -2II \\ \\ \end{matrix} \begin{bmatrix} 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{bmatrix}$$

$$x_1 = t \quad x_4 = -2r$$

$$x_2 = r \quad x_5 = r.$$

$$x_3 = s$$

$$3) \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{Note } \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{ is not in rref, There is a leading 1 in the 2nd row, but not in the row above it, the first row.}$$

$$4) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & a & b \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & a \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & b \\ 0 & 1 & a \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad a, b \text{ constants}$$

5) Suppose  $A \rightarrow B$  via elementary row operation.

a) Row  $i$  swapped with row  $j$ . To go from  $B \rightarrow A$ , just swap them back.

$$A \rightarrow B$$

b) Row  $i \rightarrow k \text{ row } i$ . To go from  $B \rightarrow A$  row  $i \rightarrow \frac{1}{k} \text{ row } i$

$$A \rightarrow B$$

c) row  $i \rightarrow \text{row } i + k \text{ row } j$ . To go from  $B \rightarrow A$  row  $i \rightarrow \text{row } i - k \text{ row } j$

Note that in each case, it is an elementary row operation.