MATH 225, Section Q1 Name:____________________________
Instructor: Michael Bush 2/22/01

# Exam 1

**Show all work, explain your reasoning and clearly mark your answers.**

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**Q1.** (14 points) Given the matrices

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 2 & 1 & 0 \\ -1 & 1 & 3 & -2 \\ 6 & 0 & 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 3 & -1 & 4 & 0 \\ 0 & 1 & 0 & 5 \\ 1 & 1 & 2 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

compute the following expressions when they are defined. When they are not defined give reasons why.

- $A - 4B$
- $B + C$
- $C^{\mathrm{T}}$
- $B\,C$
- $A\,B$
- $D^{-1}$
- $D^2$

**Q2.**

**(a)** (2 points) Write down the linear system corresponding to the vector equation

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ -6 \end{bmatrix} + x_4 \begin{bmatrix} 7 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$$

**(b)** (7 points) Use the row reduction algorithm to find the **reduced** row echelon form of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 2 \\ 0 & 1 & 3 & 1 & 3 \\ 1 & 2 & 4 & 2 & 7 \end{bmatrix}$$

**(c)** (3 points) The following matrix was obtained by applying row reduction to the augmented matrix of a linear system. Write down the **general solution** of this system, being sure to specify which variables are **free** and which are **basic**. Then express the general solution in **parametric vector form**.

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 3 \\ 0 & 0 & 1 & 4 & 2 \end{bmatrix}$$

**Q3.** (8 points) Are the columns of the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

linearly independent? Do they span $\mathbb{R}^4$? Justify both of your answers with appropriate calculations and explain your reasoning.

**Q4.** (8 points) Determine which of the following statements are true or false.

**(a)** If $A$ is the matrix with columns $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$ then $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in Span$\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$.

**(b)** Elementary matrices are always invertible.

**(c)** If $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ are linearly independent then $\mathbf{u_1}$ is in Span$\{\mathbf{u_2}, \mathbf{u_3}\}$.

**(d)** Homogeneous systems may or may not be consistent depending on whether or not there are any free variables.

**(e)** The matrix

$$\begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is in reduced row echelon form.

**(f)** Linear systems have either one solution or infinitely many solutions.

**(g)** The columns of the matrix

$$\begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & -7 & 0 & 8 \\ 0 & 5 & 1 & 1 \end{bmatrix}$$

are linearly independent.

**(h)** The matrix equation $A\mathbf{x} = \mathbf{0}$ is consistent if and only if $A$ is an invertible matrix.

**Q5.** (8 points) Determine whether or not the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -3 & 2 & 2 \\ 2 & -1 & -1 \end{bmatrix}$$

is invertible and find the inverse if it is using the row reduction method demonstrated in class.